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Frequency Analysis and System Response of a 1/18th Scale Race Car

The objective of this project is to preform experiments, gather data and model the vehicles suspension for future development of a control algorithm that would enhance the vehicles autonomous driving capabilities. Currently the car only utilizes image processing techniques to make steering and throttle decisions. The motivation for gathering this data is to create a model that approximates the response of the suspension, so that it can be incorporated into the simulation which will learn over several thousand iterations around a track. The cars goal is to complete laps around a track as fast as possible and leveraging the accelerometer data to push the car harder around corners will aid in achieving this goal.

Experiments

There are three experiments that will be performed to approximate the total stiffness and dampening coefficients for a simple quarter car lumped parameter model. The first experiment measures the static displacement of the springs under a known mass. Similarly, the total stiffness of the vehicle is measured using the height difference of the chassis to the ground. The spring and total stiffness can be approximated, which will aid in calculating the tire stiffness from the total stiffness of the vehicle measured by the accelerometer. The second experiment requires that the vehicle drive over a bump as a step input. Additionally, a bounce test was performed while the car was at rest. The accelerometer data from this experiment will provide the approximate total stiffness and dampening coefficients of the vehicle using the method of logarithmic decrement [2]. Using MATLAB, the estimated stiffness and dampening coefficients can be evaluated and compared to the accelerometer data. Finally, in the third experiment the vehicle will be driven over a sinusoidal track. The recorded response will be compared to the MATLAB model developed in the second experiment but in this case under a sinusoidal input. After the conclusion of these three experiments, the stiffness and dampening coefficients will be validated in order to provide a model that is adequate for future controls development.

The national instruments (NI) MyRIO is a compact data acquisition system (DAS) that has an onboard accelerometer which was used to record experimental acceleration data. Excel was used to integrate this data over time to get velocity and displacement data. MATLAB was used to compare response of the approximated transfer function to the measured displacements. The test track was built on a budget using a single 2x10x8 foot board and a 1” wooden rod.

Data Acquisition

Using LabVIEW to program the MyRIO DAS the block diagram in figure 1 was deployed onto the MyRIO. To record the acceleration data in the *z*-axis, a for loop is initialized by pressing the button on the MyRIO and pushed again to finish the recording. Data is saved onto a flash drive in a .CSV file, which can then be extracted and opened into excel.

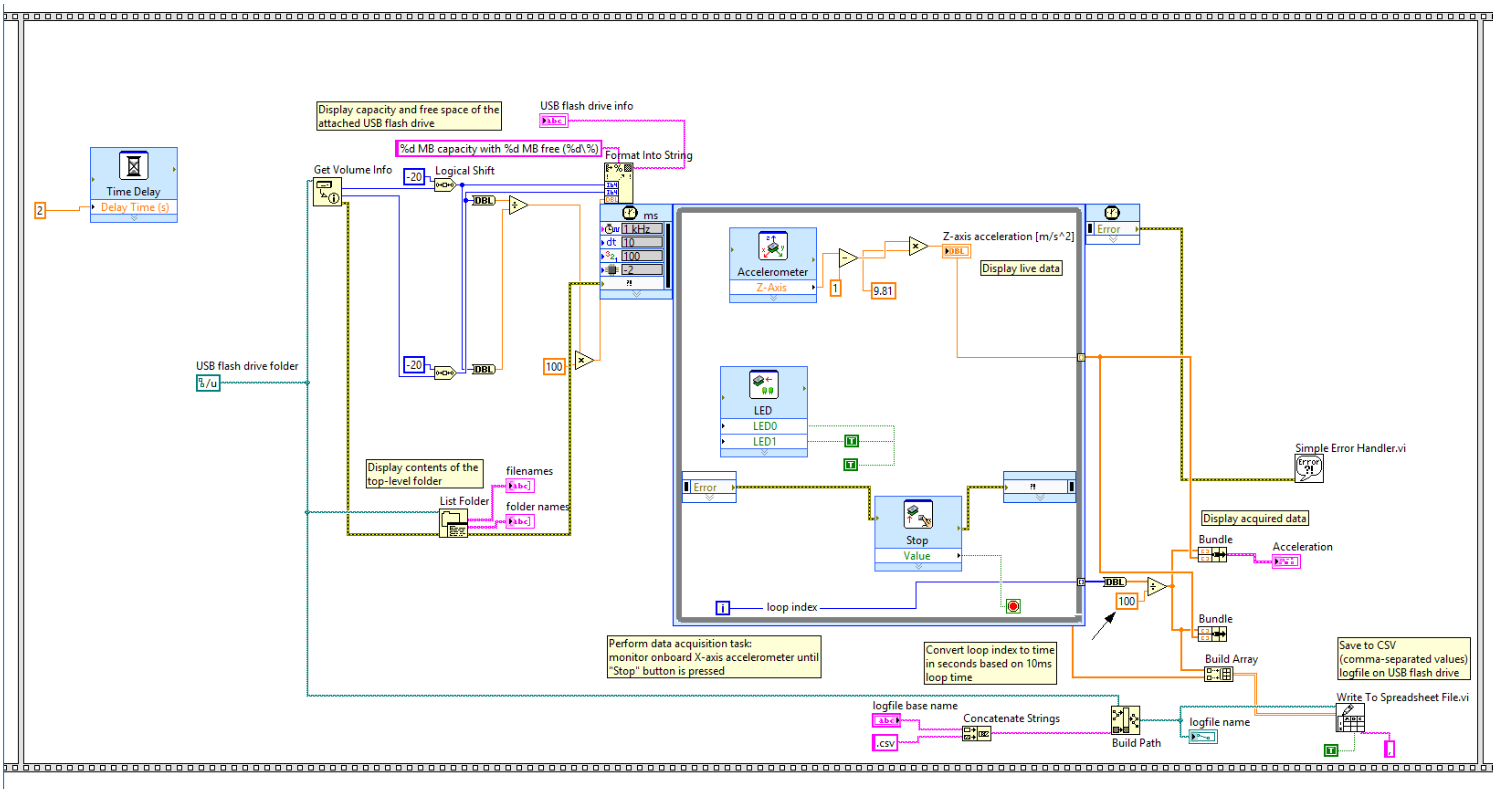

Figure 1. Block diagram *z*-axis acceleration data acquisition using NI MyRIO

The MyRIO was strapped on top of the vehicles center of gravity as seen in figure 2. Unfortunately, the power and communications cables had to be connected during testing, which made it difficult to test on longer tracks.

Figure 2. NI MyRIO with onboard accelerometer.

Test Tracks

The second experiment requires that the vehicle drive over a bump as a step input. The track for this experiment will only have one bump, with the speed of the vehicle being measured using a camera and distance markings on the track. In the third experiment the car will be driven over 3 evenly spaced 1 cm high bumps. The spacing of the bumps is 9 inches. The speed of the vehicle will again be recorded using a camera, but not held constant as it is not possible to set a specific speed on the vehicle.

Figure 3. Generic track configuration.

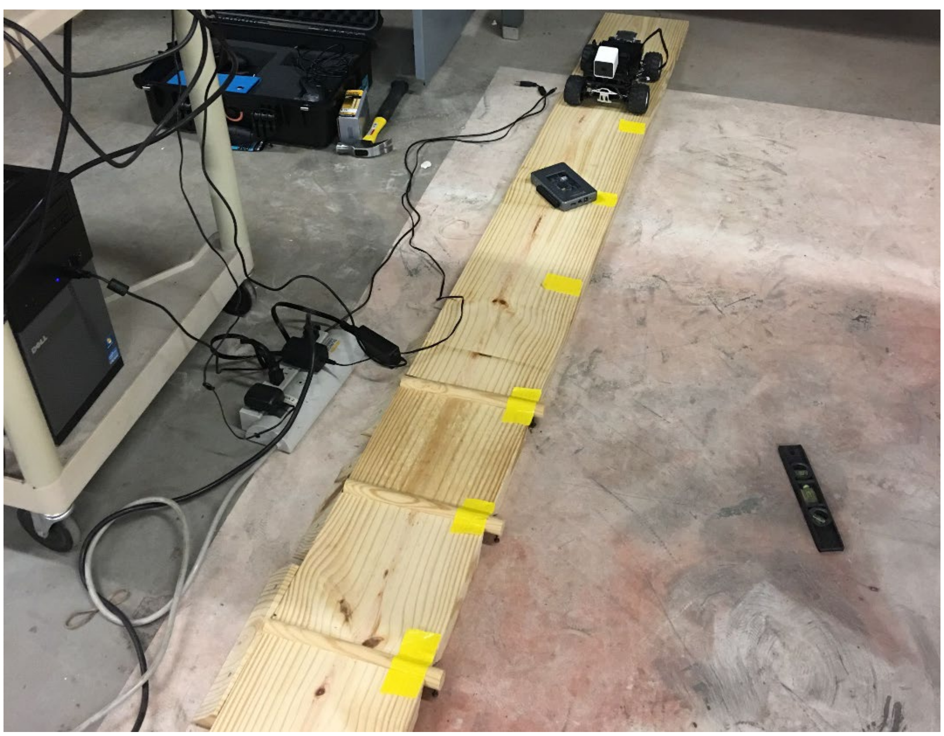

Figure 4. Sinusoidal test track for experiment 3.

Experimental Results

Static Displacement Test

In order to determine the stiffness of the springs on the car, I measured the static height of all the springs to determine an average unloaded length of the spring, x_o. Then I placed 3 different weights on the center of gravity, and measured the displacement, x_d , of the springs. I then plotted the mass as a function of the difference between the unloaded length and the displaced length as can be seen in figure 5. The vehicle has a measured mass of 1,656 grams.

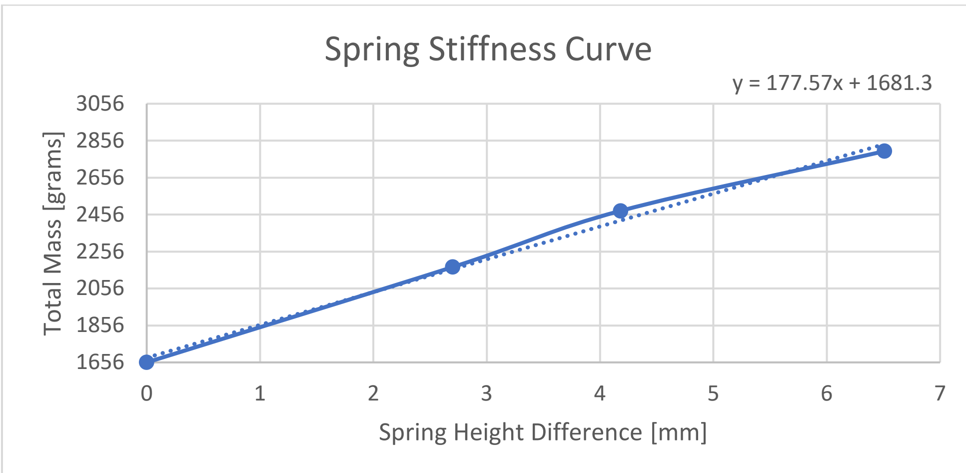

Figure 5. Spring stiffness curve

To verify the total stiffness of the vehicle, I measured the distance from the bottom of the chassis to the ground, h_o, and the displaced height under a known mass, h_d. I also placed 3 different weights on the center of gravity and measured using a depth gage on a caliper. This test will help validate results extrapolated from the Step Input Test. The mass is plotted as a function of the difference between the unloaded chassis and displaced chassis heights as can be seen in figure 6.

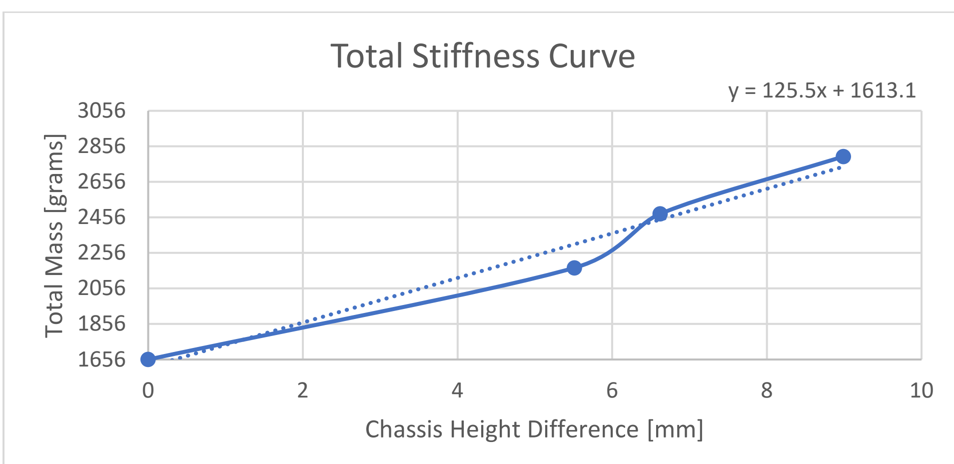

Figure 6. Total stiffness curve

Step Input Test

I preformed two different types of step input tests to try and approximate the total mass and stiffness coefficients of the vehicle. In the first step input test, I drove the vehicle over a single 1 cm high bump at a speed of 0.47 m/s. Using accelerometer data, I calculated the *z*-axis displacement over time, as can be seen in figure 7.

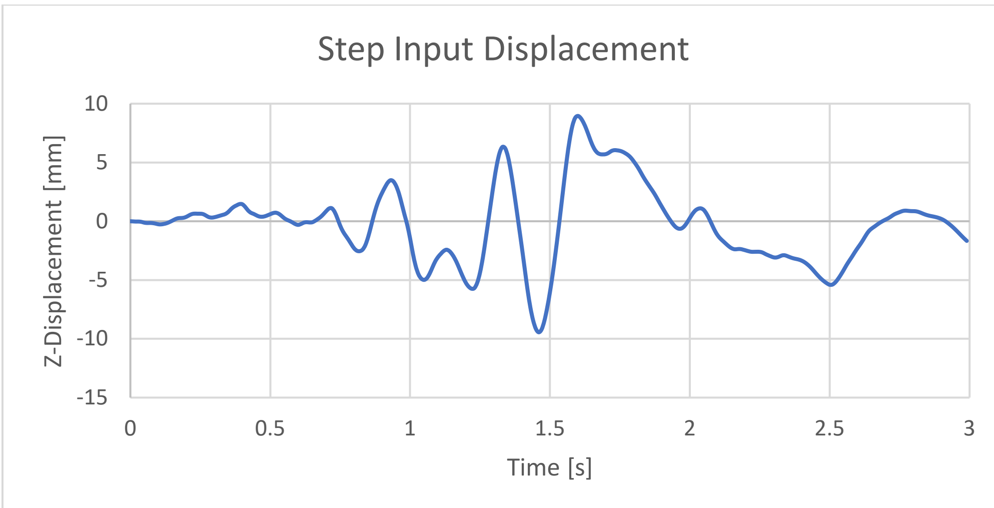

Figure 7. Step input response over a 1 cm bump at 0.47 m/s.

These results are difficult to extrapolate using the method of logarithmic decrement, because the peaks grow and then shrink over time. The vehicles front wheels go over the bump causing the front end to lift up and then at about 1.5 seconds the rear of the car hits the bump forcing the front end down. This sort of response is not quite a step input, rather it is a series of step inputs that forces the vehicle to swing backward and then quickly forward. This can be seen in video 1, which is attached as supplemental material with this report [3].

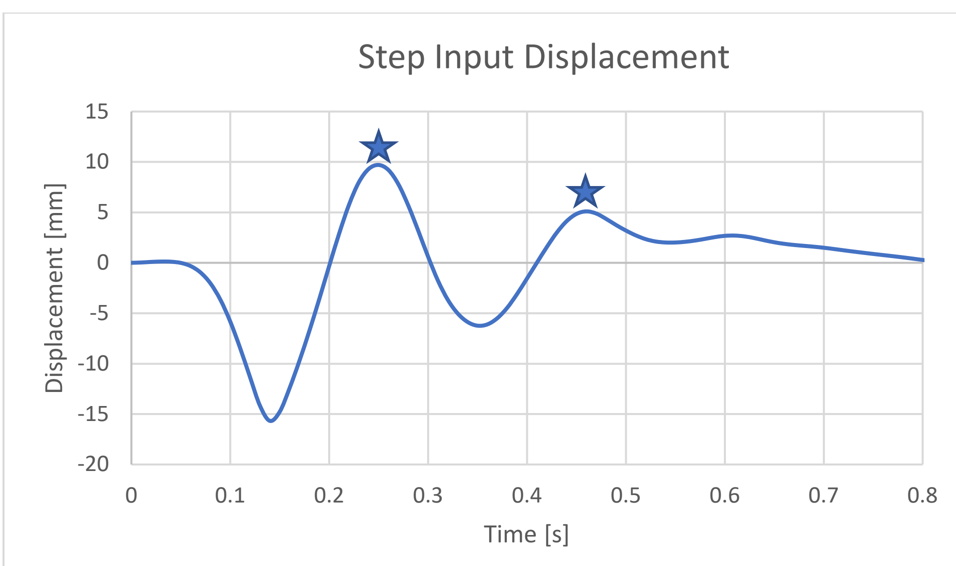

Figure 8. Step input response bouncing the vehicle

In the second step input test, the vehicle was at rest and I quickly bounced the vehicle using my finger. I calculated the displacement of the vehicle as the frequency of oscillations died out. Figure 8 shows this, and it is possible to extrapolate information about the total stiffness and dampening of the system using the method of logarithmic decrement. I included video 2 to show this test being carried out [3].

Sinusoidal Input Test

I performed several tests over 3, 1 cm high bumps that were spaced evenly 9 inches apart from each other. I had trouble getting a good data from the accelerometer, as many of my measurements had high DC offset in the results [1]. I was able to get two measurements that had negligible bias in the displacement results, and they are shown in figures 9 and 10 below. Video 3 shows the results collected in figure 9 [3].

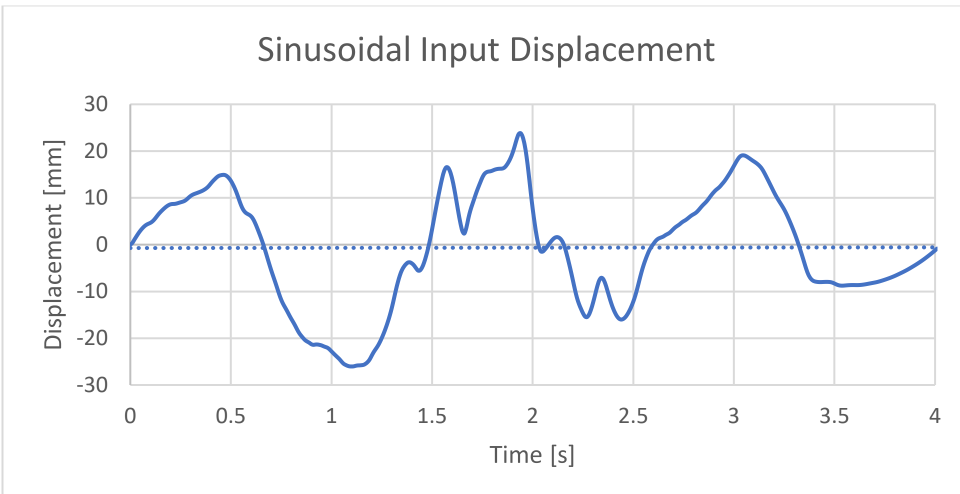

Figure 9. Sinusoidal input with the car traveling at 0.66 m/s.

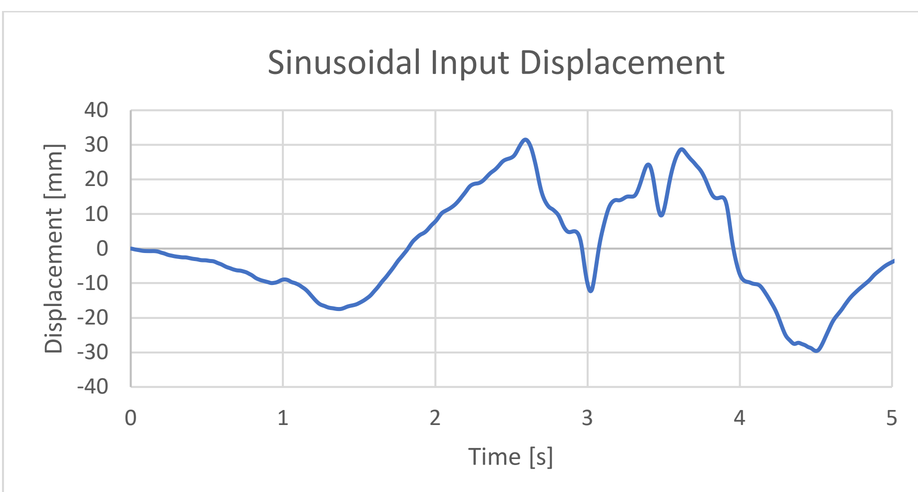

Figure 10. Sinusoidal input with the car traveling at 0.48 m/s.

Analytical Methods

Spring and Total Stiffness Approximation

The spring and total stiffness curves are approximated as linear in figure 5 and 6. The stiffness coefficient, k, is simply the slope of each of these curves. This information will be helpful in validating stiffness values derived from the step input test.

Tire Stiffness Approximation

In order to determine the tire stiffness, the total stiffness and spring stiffness must be known. After measuring these values in the first experiment, it is possible to calculate the stiffness of the tires. Assuming the tire and springs are working as springs in series, the following equation can be used to determine the tire stiffness.

$$k_{Total} = \frac{k_{tire} k_{spring}}{k_{tire} + k_{spring}}$$

Step Input Parameter Approximation

Using the method of logarithmic decrement, it is possible to determine the total stiffness and dampening coefficients for the system [2]. Figure 8 will be used to determine these coefficients. The logarithmic decrement, δ ,is defined to be the natural logarithm of the ratio of two successive amplitudes. Such that,

$$\delta = \frac{x(t)}{x(t+P)} \tag{1}$$

Where P is the period between the two successive peaks. The frequency of oscillation between the two peaks is the damped frequency ω_d . Thus, the period can be calculated as

$$P = \frac{2\pi}{\omega_d} \tag{2}$$

Noting that for a second order system, the damped frequency is also defined as:

$$\omega_d = \omega_n \sqrt{(1-\xi^2)} \tag{3}$$

Substituting the free response of the underdamped system into equation 1, we eventually obtain,

$$\delta = \xi \omega_n P \tag{4}$$

Combining equations 2, 3 and 4 we can obtain,

$$\delta = \frac{2\pi\xi}{\sqrt{1-\xi^2}} \tag{5}$$

Which can be rewritten in terms of the dampening ratio, ξ

$$\xi = \frac{\delta}{\sqrt{4\pi^2 + \delta^2}} \tag{6}$$

Once this is determined, the damping frequency can be solved for, where $t_1 - t_o$ is the period between the peaks starred in figure 8.

$$\omega_d = \frac{2\pi}{(t_1 - t_o)} \tag{7}$$

The natural frequency can be solved for from equation 3, now that both the damping frequency and dampening ratio are known. The system stiffness can be approximated using natural frequency and the measure mass of the vehicle.

$$k = \omega_n^2 m \tag{8}$$

Finally, the system dampening coefficient, c, can be approximated by solving the equation for dampening ratio,

$$c = 2\xi\sqrt{mk} \tag{9}$$

Lumped Parameter Model

As previously mentioned, the analysis of the vehicle dynamics is being modeled based on the 1 degree of freedom, quarter car model. The equations of motion for this system are

$$m\ddot{y} = -k(y-h) - c(\dot{y} - \dot{h})$$

$$m\ddot{y} + c\dot{y} + ky = c\dot{h} + kh$$

Laplace transforming the above equation, the transfer function for this system is

$$G(s) = \frac{Y(s)}{H(s)} = \frac{cs + k}{ms^2 + cs + k} \tag{10}$$

This model assumes that a quarter of the vehicles mass is on one wheel. However, Since the measured accelerations and displacements are for the entire vehicle, the mass of the block will be the mass of the entire car.

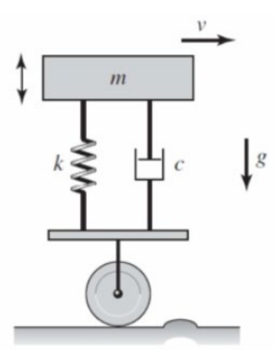

Figure 11. 1 degree of freedom quarter car model.

Sinusoidal Input Analysis

This test was performed in order to evaluate the parameters derived from the step input test. Using the transfer function for the one degree of freedom quarter car model and MATLAB's transfer function and lsim functions, a simulated response will be generated. This simulated response will be used to validate the robustness and similarity of the approximated stiffness and dampening coefficients. In order to perform this analysis, the height of the track must be developed as a input function for MATLAB. The frequency is dependent on the speed measured, as well as the distance of the bumps.

$$\omega = \frac{2\pi V}{d} \tag{13}$$

Where V is velocity, and d is the spacing of the bumps. Since the test track is not truly a sinusoidal function, a half-rectified sine wave function was used in MATLAB for lsim.

<u>Analytical Results</u>

Using the results from figure 5 and 6, the spring and total stiffness coefficients can be approximated.

- Spring Stiffness, k_{spring} : 177.57 g/mm (1,741 N/m)
- Total Stiffness, k_{Total} : 125.5 g/mm (1,231 N/m)

Using these results the tire stiffness can be approximated

- Tire Stiffness, k_{Tire} : 427.9 g/mm (4197 N/m)

Analysis of the step input from figure 8 yielded the following results

Logarithmic Decrement, δ	0.642885	
Damping Ratio, ξ	0.101787	
Damping Frequency, ω_d	26.17994	Hz
Natural Frequency, ω_n	26.31662	Hz
Total Stiffness Coefficient	1291.702	N/m
Total Dampening Coefficient	9.992045	N*s/m

Table 1. Stiffness and dampening coefficient approximation from the step input bounce test.

The total stiffness measured during the static tests is very similar to the total stiffness derived from the logarithmic decrement method. This agreement between the two methods is a good reference to check how realistic these results are. Assuming the spring stiffness is held constant from the static tests, the new tire stiffness coefficient would be 509 g/mm (4997 N/m).

Comparing the Approximated Model to Measured Results

Using the MATLAB code included in Appendix A, I created a half-rectified sin wave, with an amplitude of 10 mm. The frequency of the input function was determined using equation 13. Figure 9 from the sinusoidal experiment was used for comparison, thus the velocity of the simulated input frequency was set to 0.66 m/s.

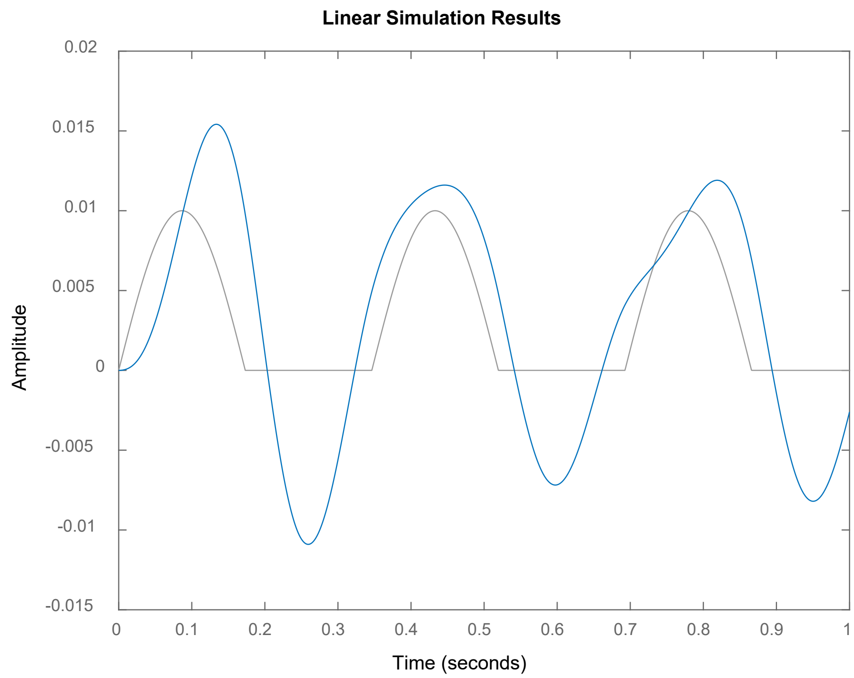

Figure 12. Simulated response, with the amplitude of displacement [m] as a function of time

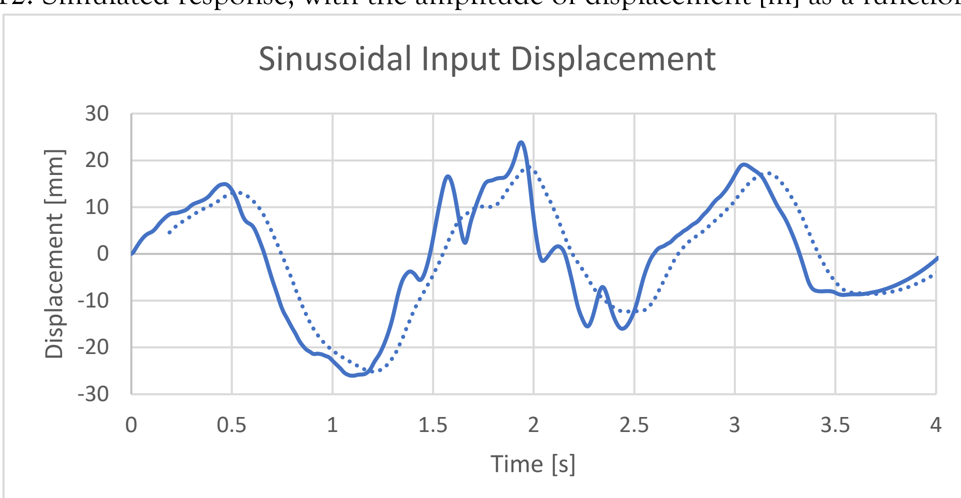

Figure 13. Experimental sinusoidal input with the car traveling at 0.66 m/s.

Although these two curves peak at different curves, it is hard to argue that they are not similar. There is noise in the displacement measurement, however when a moving average is approximated over this noise it fits almost exactly to the simulated response of the system.

Conclusion

This project was a valuable learning experience, introducing me to new methods of measurement, analysis and engineering. Gathering the data was challenging, as I have had limited experience coding in LabVIEW, but I was able to learn a lot about the process through this project. Dr. Beasley and Dr. Figliola's mechanical measurements book [1] was very helpful in understanding what DC offset is and why is occurs in signal processing. Being able to integrate the research done in my creative inquiry with the course material was exciting and a great way to get a better grasp of the underlying concepts covered in modeling dynamic systems. I taught myself some of the methods needed to extract the stiffness and dampening coefficients from the accelerometer data [2]. As expected, not everything went smoothly during the process of creating an empirical model that closely resembles observations made in reality. Despite this, I think that the approximation methods used here provided realistic and useful results.

References

[1] "Static and Dynamic Characteristics of Signals." *Theory and Design for Mechanical Measurements*, by R. S. Figliola and Donald E. Beasley, Wiley, 2015.

[2] "Parameter Estimation in the Time Domain." *System Dynamics*, by William J. Palm, McGraw-Hill Science, 2014, pp. 473–516.

[3] Videos are available at this link: https://spark.adobe.com/video/byEgTq6KgdgB1

Appendix A

MATLAB Code for Simulated Response

```
V=0.66 %m/s
d=.2286 %m
w=2*pi*(V)/d

% Half-Wave Rectified Sine Function
f = @(t) .01.*sin(w.*t).*(.01.*sin(w.*t)>=0) + 0*(.01.*sin(w.*t)<0);
t = linspace(0, 1,1000);

m = 1.865 %kg mass of car and MyRIO
c = 10 %N*s/m
k = 1291 %N/m

Gs = tf([c k],[m c k])
in = f(t)
lsim(Gs,in,t)
```